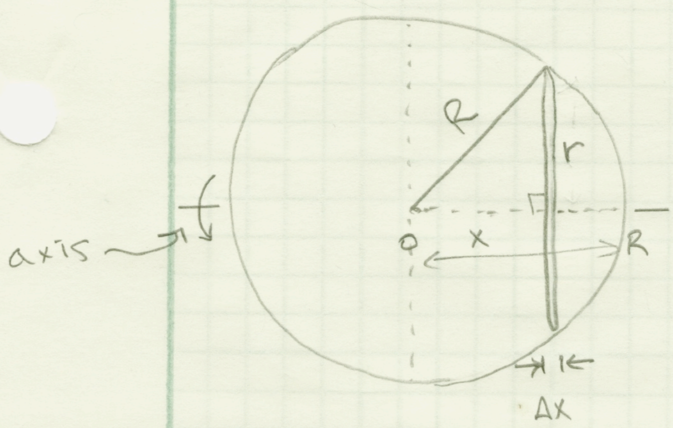
MOMENT OF INERTIA PROOF (SOLID SPHERE ROTATING ABOUT CENTER)

(X-AXIS HERE)



thickness of disk

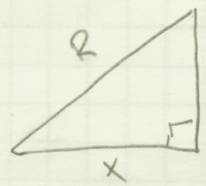
SOLID DISKS TECHNIQUE

Imagine the sphere can be sliced vertically into infinitely thin pieces

R = radius of sphere

x = distance to "disk"

r = radius of disk



$I = \int_0^R r^2\,dm$ → we need r, from PYTHAG.

$r^2 = R^2 - x^2$

$I = \int_0^R r^2\,dm$ → but, since $m = \rho V$ (disk of thickness Δx)

$m = \rho \frac{1}{2} \pi r^2 (\Delta x)$

So $dm = \rho \frac{1}{2} \pi r^2\,dx$

$I = \int_0^R r^2 \left(\rho \frac{1}{2} \pi r^2\,dx\right) = \rho \frac{1}{2} \pi \int r^4\,dx$ → remember $r^2 = R^2 - x^2$

(realize the limits $0 \to R$ only give half the sphere, so I must double total)

so, $I = \rho \frac{1}{2} \pi \int (R^2 - x^2)^2\,dx = \rho \frac{1}{2} \pi \int R^4 - 2R^2x^2 + x^4\,dx$

$I = \rho \frac{1}{2} \pi \left[R^4 x - \frac{2R^2x^3}{3} + \frac{x^5}{5} + C\right]\Big|_0^R$ (then x 2 b/c of limits selected)

NOW WE PLUG IN OUR LIMITS AND EVALUATE.

REMEMBER THIS IS ALWAYS (FINAL - INITIAL)

I ALSO COULD HAVE SET MY LIMITS TO -R to R cont.)

$$I = \rho \tfrac{1}{2} \pi \left[R^4 x - \frac{2R^2 x^3}{3} + \frac{x^5}{5} + c \right] \Big|_0^R$$

$$I = \rho \tfrac{1}{2} \pi \left[R^4 (R) - \frac{2R^2 R^3}{3} + \frac{R^5}{5} \right]$$

only half the sphere

$$I = \rho \tfrac{1}{2} \pi \left[R^5 - \tfrac{2}{3} R^5 + \tfrac{1}{5} R^5 \right]$$

$$I = \rho \tfrac{1}{2} \pi \left[\frac{8}{15} R^5 \right]$$

$$I = \rho \tfrac{1}{2} \pi \left[\frac{8}{15} R^5 \right] (\times 2)$$

double total to get both halves of sphere

$$I = \rho \pi \left(\frac{8}{15} \right) R^5$$

$$\rho = \frac{m}{V} \leftarrow \text{sphere}$$

$$I = \left(\frac{m}{\frac{4}{3} \pi R^3} \right) \pi \left(\frac{8}{15} \right) R^5$$

$$\rho = \frac{m}{\frac{4}{3} \pi r^3}$$

Sub into

$$I = m \frac{24}{60} R^2$$

$$\boxed{I = \frac{2}{5} m R^2}$$

SIMPLE.

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